

Stability of P_{11} Resonances Extracted from πN data

[Phys. Rev. C **81**, 065207 (2010)]

Satoshi Nakamura

(Excited Baryon Analysis Center, JLab)

Collaborators

H. Kamano (Osaka City U.)

T. S.-H. Lee (ANL)

T. Sato (Osaka U.)

Introduction

Extraction of N^* from πN data is important !

- * Understanding spectrum and structure of N^* within QCD

Steps to extract N^*

1. Construct a reaction model through analysis of data
2. From the constructed model, resonance properties (pole position, vertex form factor) are extracted with analytic continuation

Extraction of N^* properties is inevitably model-dependent !

Several approaches : dynamical model (EBAC, Jülich ...)

 K-matrix (GWU/VPI, Bonn-Gatchina ...)

Existence of some N^* is controversial

Question !

How much extracted N^* parameters depend on :

1. model
2. precision of data (amplitude)

We study stability of pole structure :

Roper resonance [$N(1440)P_{11}$]

- * Large variation of parameters of EBAC-DCC model
- * Including bare nucleon state
- * Variation of P_{11} amplitude for $1.6 < W < 2$ GeV

Higher mass P_{11} resonances

- * Fit to amplitude up to $W \sim 2.5$ GeV
- * Simultaneous fit to πN , ηN , $\pi \Delta$ channels

EBAC-DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007)

Coupled-channel Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

$$\{a, b, c\} = \pi N, \eta N, \pi\pi N (\pi\Delta, \sigma N, \rho N)$$

EBAC-DCC (Dynamical Coupled-Channel) model

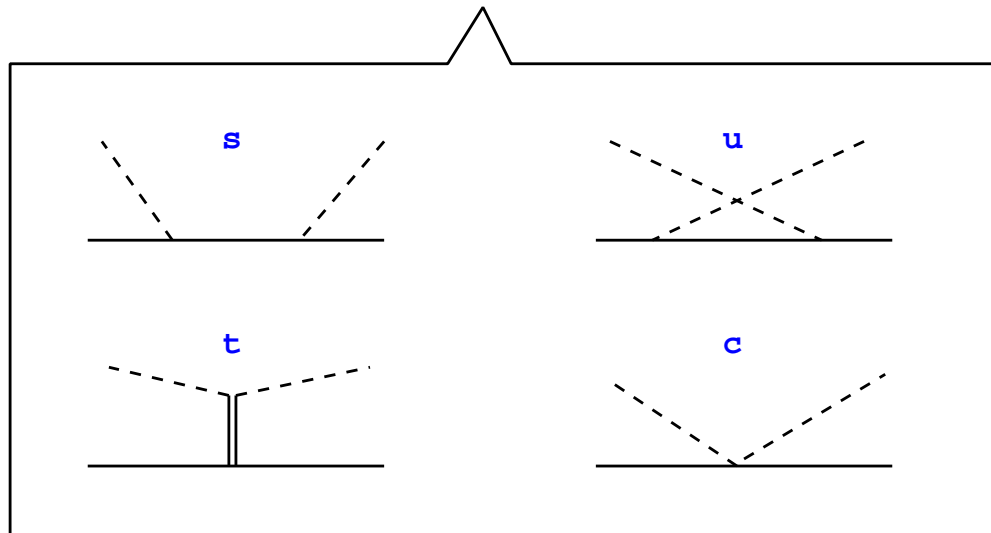
Matsuyama et al., Phys. Rep. **439**, 193 (2007)

Coupled-channel Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

$$V_{ab} = \text{[diagram 1]} + \text{[diagram 2]}$$

The diagram shows the potential V_{ab} as the sum of two terms. The first term is a vertex with a solid line and a dashed line meeting at a black dot. The second term is a vertex with a solid line and a dashed line meeting at a point, with a horizontal line segment labeled "bare N*" between them.



Bare Nucleon Model

Pearce and Afnan, PRC **34**, 991 (1986)

Motivation : P_{11} amplitude of EBAC-DCC model below πN threshold, $E \sim m_N$

EBAC-DCC

$$T \sim \Gamma_{\pi NN}^* \left[\frac{d_1}{E - m_N} + \frac{d_0}{(E - m_N)^2} + d_2 \right] \Gamma_{\pi NN}$$

Jülich

$$T \sim \Gamma_{\pi NN}^* \left[\frac{d'_1}{E - m_N} \right] \Gamma_{\pi NN}$$

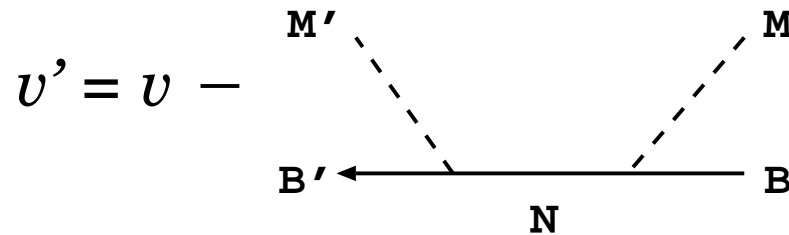
Question : Is Roper poles sensitive to the analytic structure at $E \sim m_N$?

Bare Nucleon Model

Pearce and Afnan, PRC **34**, 991 (1986)

* Potentials

$$V_{ab} = v'_{ab} + \frac{\Gamma_{N,a}^\dagger \Gamma_{N,b}}{E - m_N^0} + \sum_{N^*} \frac{\Gamma_{N^*,a}^\dagger \Gamma_{N^*,b}}{E - m_{N^*}^0}$$



* Nucleon Pole Condition

$$(i) \quad T(E \sim m_N) \sim \frac{\bar{\Gamma} \bar{\Gamma}}{E - m_N}$$
$$(ii) \quad \bar{\Gamma} = F_{\pi NN}^{\text{phys}}$$

Extraction of N^* information with analytic continuation

Suzuki et al., PRC **79**, 025205 (2009)

πN scattering amplitude near a pole ($E \sim M_R$)

$$F_{\pi N}(E) \sim \frac{\bar{\Gamma}(M_R) \bar{\Gamma}(M_R)}{E - M_R} + (\text{regular terms})$$

Parameters characterizing Resonance

- * Pole position of amplitude : M_R
- * $N^* \rightarrow MB$ decay vertex : $\bar{\Gamma}(M_R)$

Results

Benchmarks

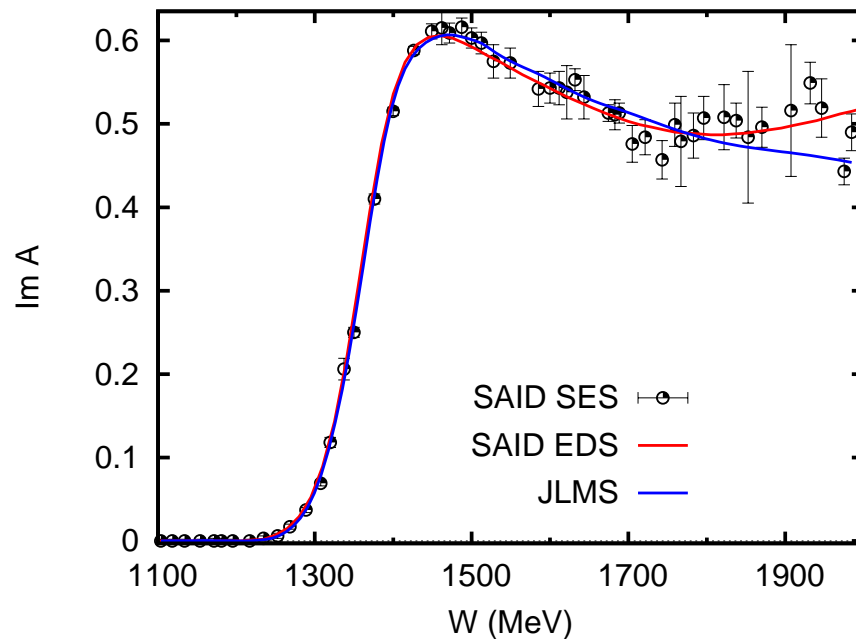
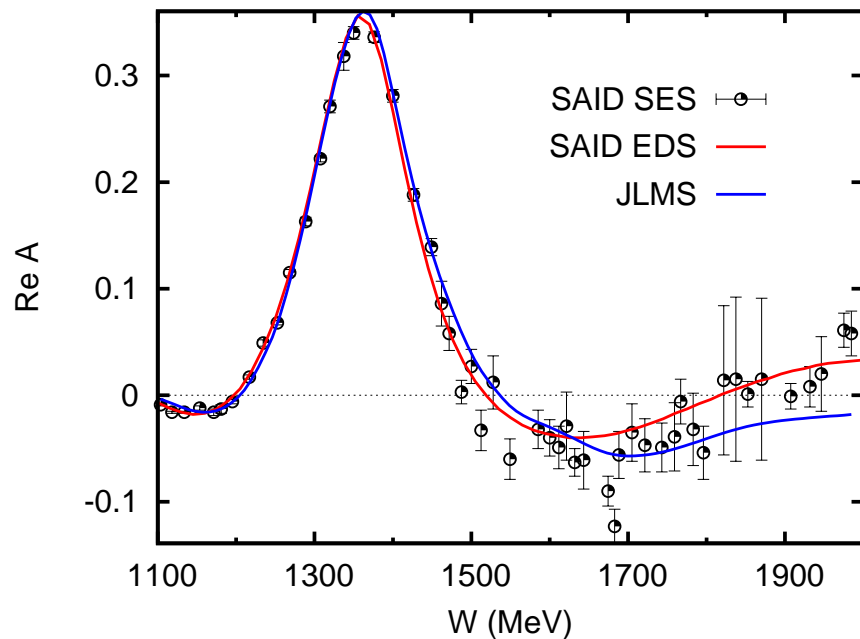
- * SAID SES (single energy solution) (SP06)

⇐ Parameters of all models are fitted

- * SAID EDS (energy-dependent solution) (SP06)

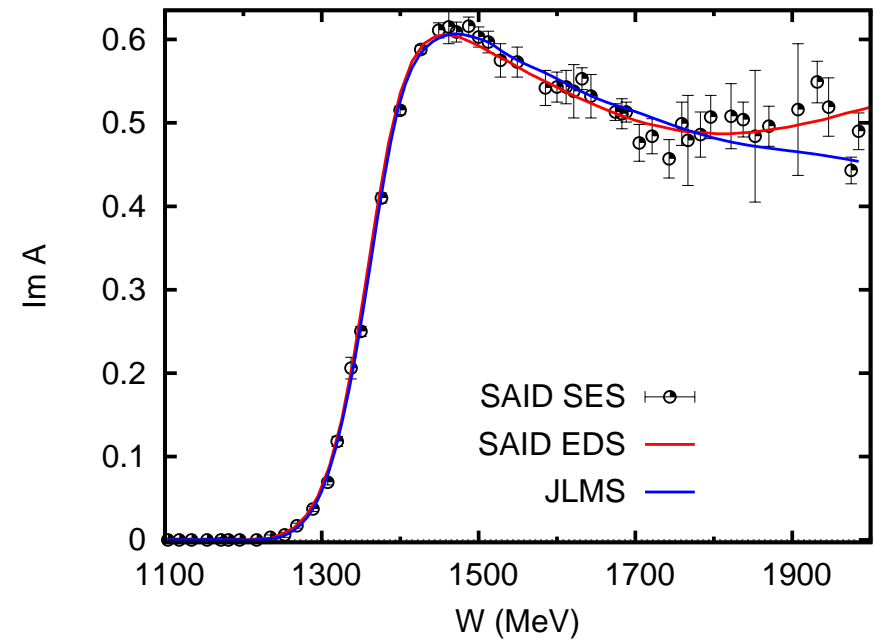
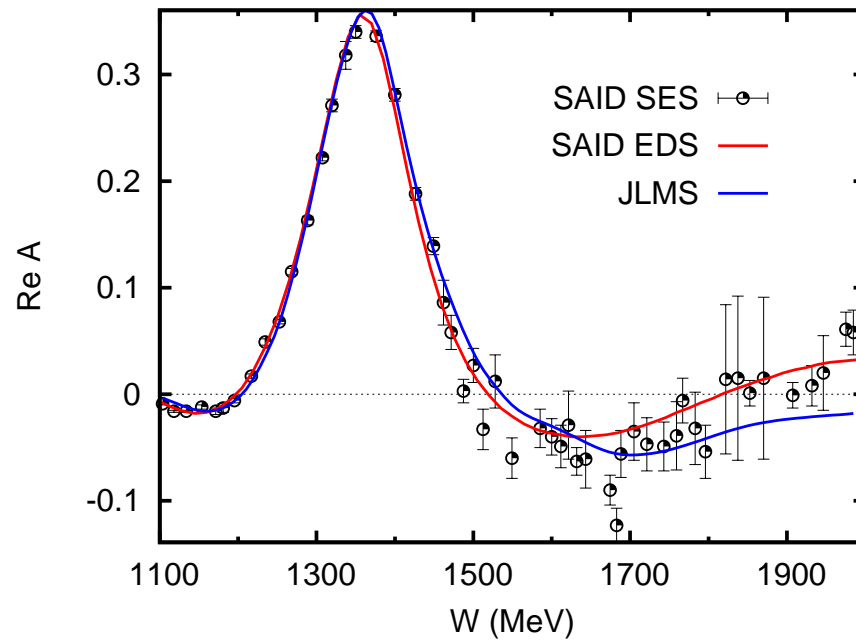
- * JLMS (Julía-Díaz et al., PRC **76**, 065201 (2007))

* P_{11} amplitude (benchmarks)



Model	$upuupp$	$upuppp$	$uuuuup$	$\chi^2 / (\# \text{ of data})$
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	2.94
JLMS	(1357, -76)	(1364, -105)	(1820, -248)	3.55

* P_{11} amplitude (benchmarks)



* Sheet assignment of the poles

$$(s_{\pi N}, s_{\eta N}, s_{\pi\pi N}, s_{\pi\Delta}, s_{\rho N}, s_{\sigma N}) = (u, p, u, u, p, p)$$

More Models 1

* $2N^*-3p$

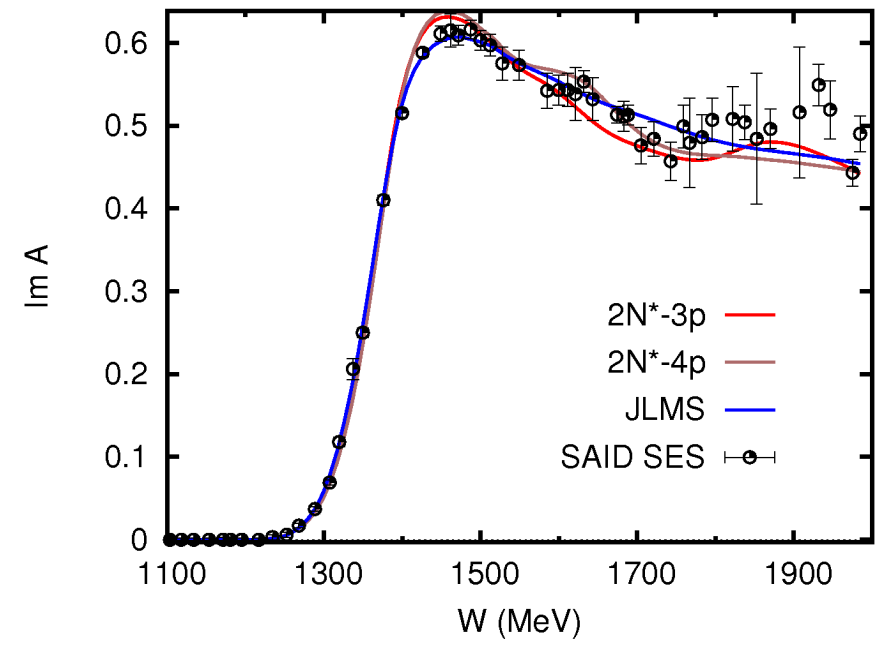
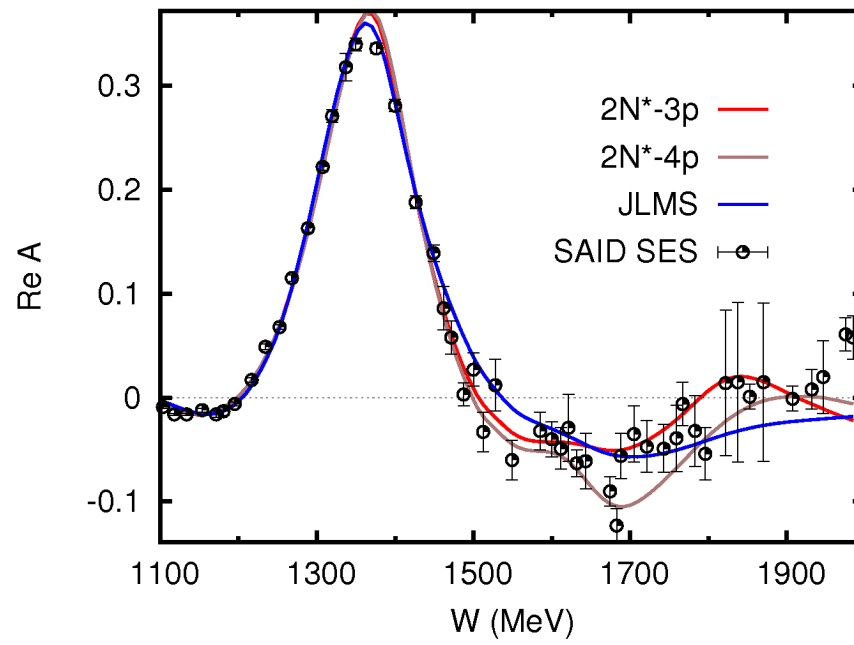
$v \neq v_{JLMS}$

smooth fit

* $2N^*-4p$

$v \neq v_{JLMS}$

fitted to oscillated behavior



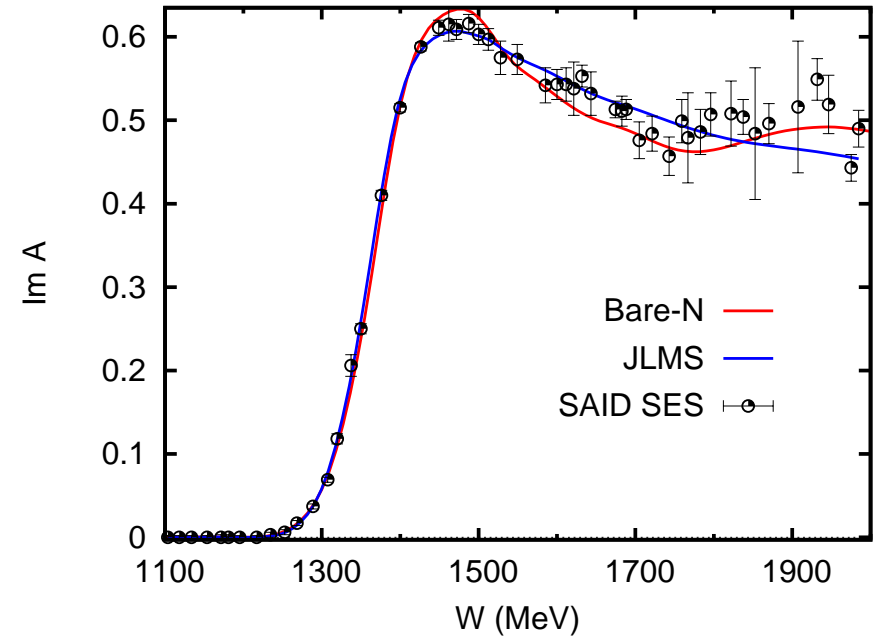
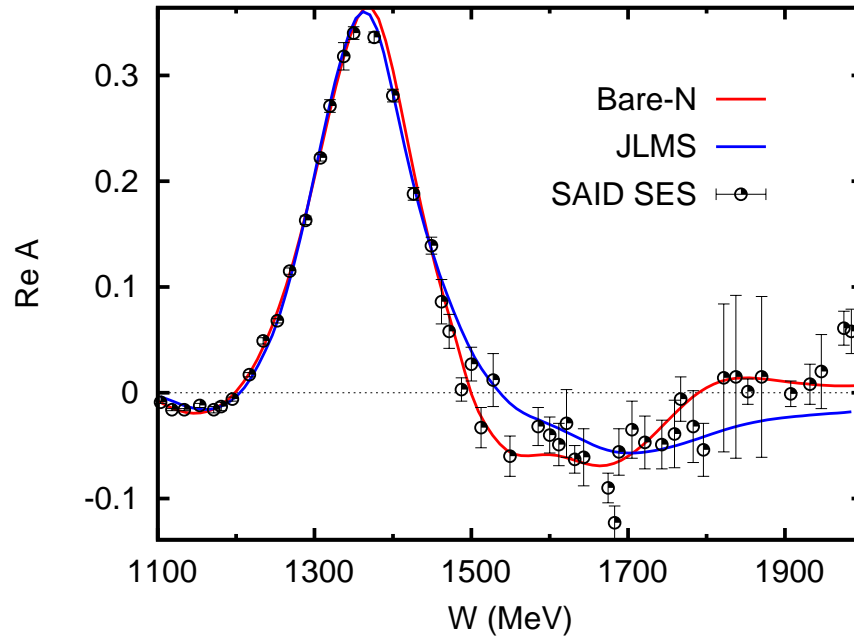
Model	$upuupp$	$upuppp$	$uuuupp$	$uuuuup$	χ^2
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	—	2.94
JLMS	(1357, -76)	(1364, -105)	—	(1820, -248)	3.55
$2N^*-3p$	(1368, -82)	(1375, -110)	—	(1810, -82)	3.28
$2N^*-4p$	(1370, -81)	(1384, -115)	(1635, -68)	(1960, -214)	3.36

* Roper two poles are stable !

* Additional pole in $2N^* - 4p$!

* Different high W amplitude \Rightarrow Different higher mass pole position

More Models 2 (Bare Nucleon Model)



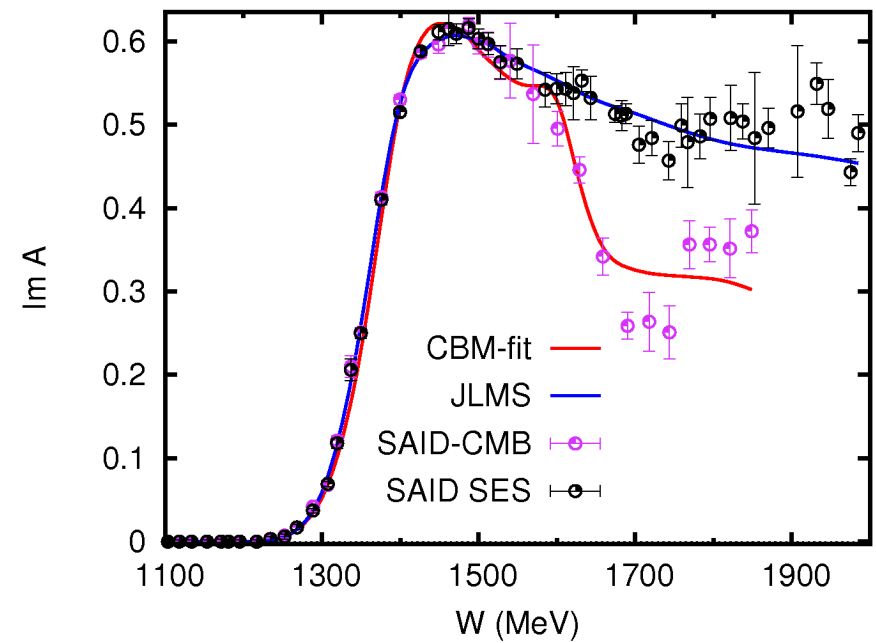
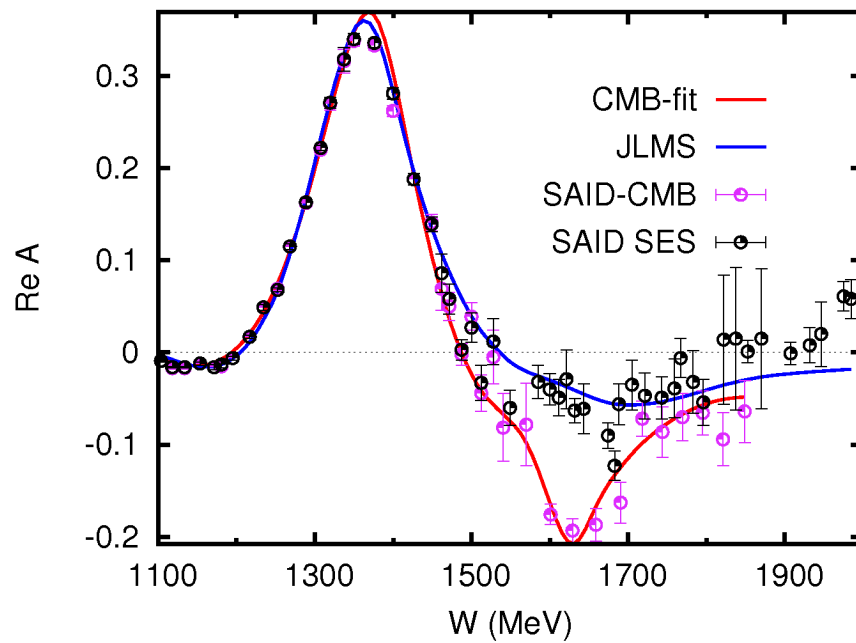
Model	$upuupp$	$upuppp$	$uuuupp$	$uuuuup$	χ^2
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	—	2.94
JLMS	(1357, -76)	(1364, -105)	—	(1820, -248)	3.55
$1N_01N^*-3p$	(1364, -81)	(1377, -129)	—	(1769, -132)	2.51

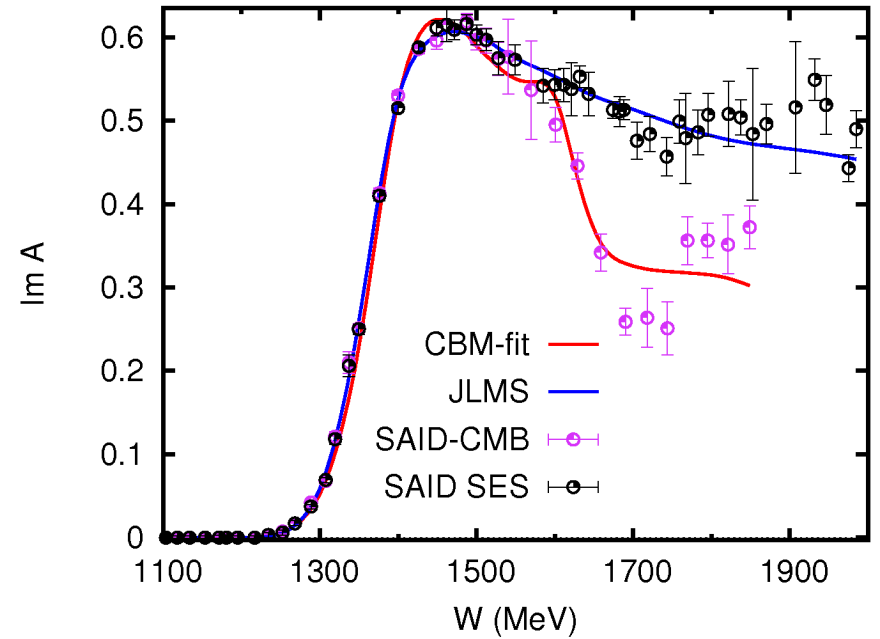
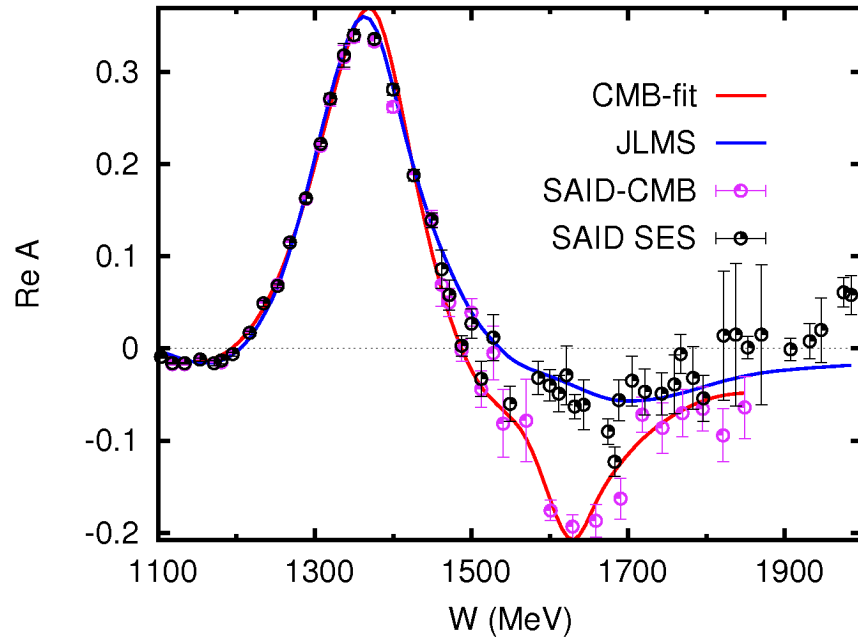
* Roper two poles are stable, not sensitive to the analytic structure at $E \sim m_N$

Fit to Different data

* SAID-SES for $W \leq 1.55$ GeV

* CMB for $W \geq 1.55$ GeV





Model	$upuupp$	$upuppp$	$uuuupp$	$uuuuup$	χ^2
SAID-EDS(SP06)	(1359, -81)	(1388, -83)	—	—	2.94
JLMS	(1357, -76)	(1364, -105)	—	(1820, -248)	3.55
$2N^*$ -4p-CMB	(1379, -89)	(1386, -109)	(1613, -42)	(1913, -324)	4.91

* Roper two poles are stable, not sensitive to the amplitude in $W > 1.5$ GeV !

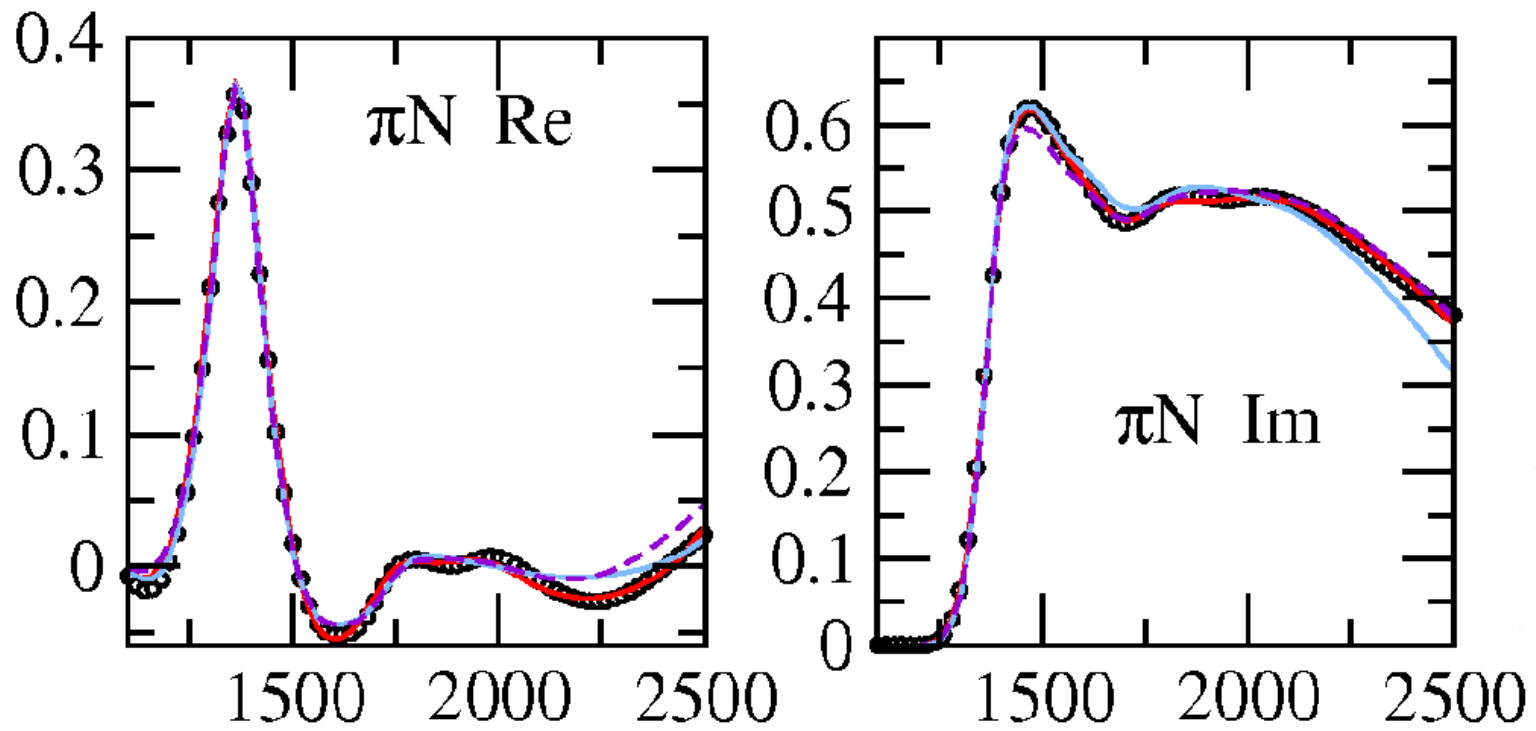
Fit up to high W & multi-channel $\pi N \rightarrow MB$

Question

- * Help stabilize higher mass pole positions ?
- * Simultaneous fit to multi-channel amplitudes
($\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$, $\pi N \rightarrow \pi \Delta$...) helps ?

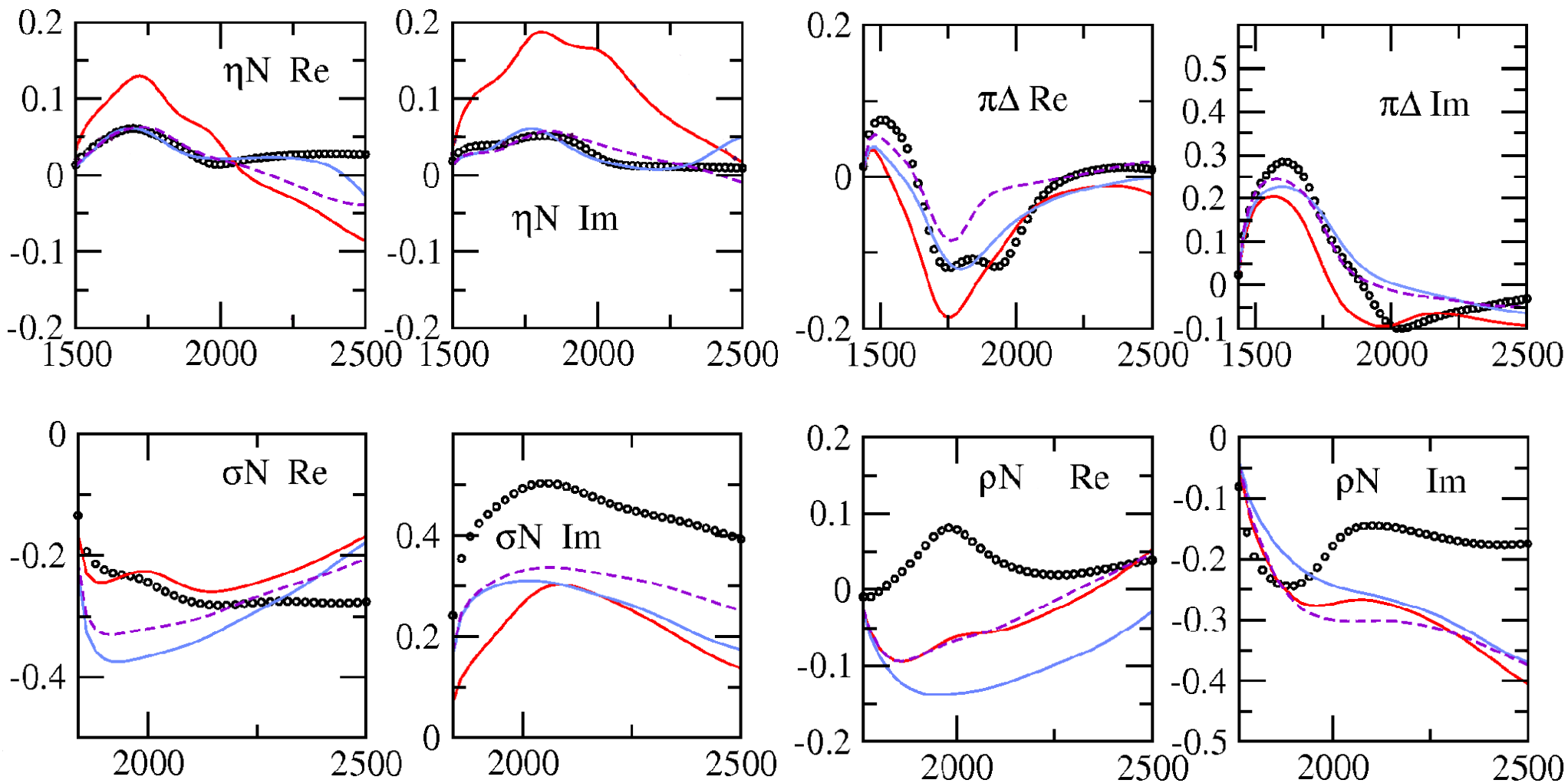
cf Ceci et al, PRL 97, 062002 (2006)

Fit to $\pi N \rightarrow \pi N$ up to $W \sim 2.5$ GeV



* 4 models, fitted to πN amplitude up to 2.5 GeV

Simultaneous fit to multi-channel $\pi N \rightarrow MB$ amplitudes



Model	$upuupp$	$upuPPP$	$uuuupp$	$uuuuup$	$uuuuuu$
black	(1360, -87)	(1368, -116)	—	(1702, -93)	(1967, -110)
blue	(1367, -80)	(1378, -101)	(1554, -68)	(1810, -179)	—
purple	(1367, -76)	(1381, -103)	(1542, -44)	(1798, -109)	—
red	(1364, -86)	(1375, -106)	—	(1778, -142)	(1999, -138)

- * Roper two poles are stable
- * Higher mass poles are still not stabilized
- * Pole $W \sim 1550 - 50i$ has very small coupling to πN

Summary

Stability of P_{11} poles

- * **Roper two poles are stable** against
 - Large variation of parameters within EBAC-DCC ($v \neq v_{JLMS}$)
 - Inclusion of bare nucleon state (different analytic structure at $E \sim m_N$)
 - Fitting to different amplitude

provided amplitudes are precisely fitted to SAID SES for $W < 1.5$ GeV

$$E_{\text{pole}} = 1363_{-6}^{+6} - i79_{-3}^{+3} \text{ (} s_{\pi\Delta} = u \text{)} , \quad 1373_{-10}^{+10} - i114_{-9}^{+15} \text{ (} s_{\pi\Delta} = p \text{)}$$

Summary

Fit to higher W & simultaneous fit to $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$, $\pi N \rightarrow \pi \Delta$

- * Roper two poles are stable

- * Higher mass poles are still unstable

\Rightarrow amplitudes of all channels need to be fitted ?